

# Slenderness Limitation in the Design of Unbraced & Braced Columns by Comparing EBCS 2: 1995 and ES EN 1992-1-1:2015

Fikadie Alamirew Alemu,

Master of Science in Civil Engineering (Structural Engineering), is a faculty member of Debre Markos University, Debre Markos Institute of Technology, Civil Engineering Academic program, Ethiopia. E-mail: [fikadiea@gmail.com](mailto:fikadiea@gmail.com) , [fikadie\\_alamirew@dmu.edu.et](mailto:fikadie_alamirew@dmu.edu.et)

**Abstract**— The codes in use in this paper are those from the EBCS 2: 1995 (Ethiopian Building Codes of Standard), and that of ES EN 1992-1-1:2015 (Ethiopian Standard-based European Norm). Typically, columns classified as short or slender based on their slenderness ratio, and this in turn affects their mode of failure. The two codes use different procedures to calculate the effective length factor and limiting slenderness ratio. This leads to different classification for a given column according to EBCS 2: 1995 and ES EN 1992-1-1:2015. The paper compares column classification, limiting slenderness ratio, and variation of effective length factors as per EBCS 2, 1995, and ES EN 1992-1-1:2015. Ten different types columns with a variation 2m height for one internal and two externals were analysed to compare its. A conclusion was drawn after investigating two-dimensional frames of two-story, with the same load applied for unbraced and braced frames. Both Results' internal and external columns have been compared and reported.

**Index Terms**— Column classification, EBCS 2: 1995, Effective length factor, ES EN 1992-1-1:2015, Slenderness ratio.

## 1 INTRODUCTION

There are different modes of failures in columns [6]. Many codes of practice rely on the effective length method to assess the stability of frames [7]. The effective length method allows the buckling capacity of a member in a structural system to be calculated by considering an equivalent pin ended column in Euler [7][6][1]. The column may fail due to concrete material crushing with negligible lateral deflection or intensified lateral deflection and spare moment leads to instability [6][1]. Hence the column design, classification plays a major role as it affects the design in later stages. Due to this effective length concepts are followed by using codes to limit slenderness ratio.

## 2. EFFECTIVE LENGTH FACTOR

Many codes of practice rely on the effective length method to investigate the stability of frames. The effective length method assess the buckling capacity in a structural system to be calculated by considering an equivalent pin ended column in Euler buckling [7]. The slender or long columns may fail due to less load when the sudden lateral displacement of the member takes place at ends. The effective length method is the most common method which allows the buckling capacity of member to be evaluated by an equivalent pin ended column using Euler buckling formulae [6][7][1]. The buckling load is termed Euler load as Euler in 1744 first obtained the value of critical load for various support conditions [2][5].

The degree of finiteness of a member at each end is calculated by considering the relative stiffness of frame elements [6]. The same concept is used in EBCS 2: 1995 and ES EN 1992-1-1:2015 to find effective length factor (k). In ES EN 1992-1-1:2015, relative end stiffness is estimated by assuming any column under consideration does not contribute anything to the rotational restraint of joint. Comparing EBCS 2: 1995 and ES EN 1992-1-1:2015 observed that the determination of effective length ratios differs from each other.

## SLENDERNESS RATIO VS. LIMITING SLENDERNESS RATIO

The ratio of effective length to the radius of gyration of the gross cross-section of the column is termed as a slenderness ratio [6]. Every code states specifically about slenderness ratio beyond which column design is affected by second-order effects. Design provision of EBCS 2: 1995 and ES EN 1992-1-1:2015 shows slenderness ratio, effective length factor, and column classification. It also compares the limiting slenderness ratio of columns as per EBCS 2: 1995 and ES EN 1992-1-1:2015. Both codes show a tedious process for determining slenderness limit  $l_{lim}$  for column classification.  $l_{lim}$  is estimated by considering creep, a quantity of steel, design moments, and axial loads in the column. Ultimately EBCS 2: 1995 and ESEN 2:2015 defines their methodology to find effective length factor and limiting slenderness ratio to get the column classification. This research focuses on this issue by taking ten different columns on the internal and edge. The effective length factor and column classifications by EBCS 2: 1995 and ES EN 1992-1-1:2015 were compared and also braced, and unbraced columns are investigated.

## 3. DESIGN PROVISIONS

The following is a summary of the steps followed for Code procedure to determine Column classification and Effective length factors according to EBCS 2:1995 and ES EN 1992-1-1:2015.

### 3.1. Effective Length Factor According to EBCS 2:1995

$$\begin{aligned} &\text{for braced non-sway frame} \\ &le/l = (am+0.4)/(am+0.8) \geq 0.7 \quad (1a) \\ &\text{For unbraced sway frame [4]} \\ &le/l = \sqrt{(7.5+4(a_1+a_2) +} \end{aligned}$$

$$1.6a_1a_2 / (7.5 + a_1 + a_2) \geq 1.15 \quad (2a)$$

Or conservatively

$$l_e / I = \sqrt{(1 + 0.8am)} \geq 1.15$$

where:

$$a_1 = (k_1 + k_c) / (k_{11} + k_{12}); a_2 = (k_2 + k_c) / (k_{21} + k_{22});$$

$$a_m = (a_1 + a_2) / 2$$

$k_1$  and  $k_2$  are column stiffness coefficients ( $EI/L$ )

$k_c$  is the stiffness coefficient ( $EI/L$ ) of the column is designed

$k_{ij}$  is the effective beam stiffness coefficient ( $EI/L$ )

= 1.0 opposite end elastically or rigidly restrained

= 0.5 opposite end free to rotate

= 0 for a cantilever beam

### 3.2. Effective Length Factor According to ES EN 1992-1-1:2015

for braced non-sway frame, [3]

$$l_o / I = 0.5 \sqrt{(1 + k_1 / (0.45 + k_1)) * (1 + k_2 / (0.45 + k_2))} \quad (1b)$$

For unbraced sway frame, [3]

$$l_o / I = \max \{ \sqrt{(1 + 10 * k_1 k_2 / (k_2 + k_1))}, (1 + k_1 / (1 + k_1)) * (1 + k_2 / (1 + k_2)) \} \quad (2b)$$

where:

$k_1, k_2$ - relative flexibilities of rotational restraints at ends 1 and 2.

$$k_1, k_2 = EI / I_{column} / \sum 2EI / I_{beam} \quad (2c)$$

### 3.3. Slenderness Ratio According to EBCS 2:1995

$$l = l_e / i \quad [4] \quad (3a)$$

$l_e$  is the effective buckling length

$i$  is the minimum radius of gyration of the concrete section only.

$$i = d / \sqrt{12} \quad (4a)$$

### 3.4. Slenderness Ratio According to ES EN 1992-1-1:2015

$$l = l_o / I \text{ where } l_o - \text{effective length} [3] \quad (3b)$$

$$i = \text{Radius of gyration} = \sqrt{I / A} \quad (4b)$$

### 3.5. Column classification According to EBCS 2:1995

for braced non-sway frame (Development, 1995)

$$\text{Short if } l \leq 50 - 25(M_1 / M_2) \quad (5a)$$

$$\text{Slender if } l > 50 - 25(M_1 / M_2) \quad (6a)$$

for un braced sway frame [4]

$$\text{Short if } l \leq \text{greater of } \{25, 15 / \sqrt{v_d}\} \quad (7a)$$

$$\text{Slender if } l > \text{greater of } \{25, 15 / \sqrt{v_d}\}, \quad (8a)$$

Where:

$$v_d = N_{sd} / (f_{cd} * A_c)$$

$f_{cd}$  = design compressive strength of concrete,  $A_c$  = cross sectional area of concrete

### 3.6. Column classification According to ES EN 1992-1-1:2015

for braced non-sway frame [3]

$$\text{Short if } l \leq 20ABC / \sqrt{n} \quad (5b)$$

$$\text{Slender if } l > 20ABC / \sqrt{n} \quad (6b)$$

for un braced sway frame [3]

$$\text{Short if } l \leq 20ABC / \sqrt{n} \quad (7b)$$

$$\text{Slender if } l > 20ABC / \sqrt{n} \quad (8b)$$

Where:

$$A = 1 / (1 + 0.2\phi_{ef}) \text{ (If } \phi_{ef} \text{ is not known, } A = 0.7 \text{ used)}$$

$$B = \sqrt{1 + 2\omega} \text{ (If } \omega \text{ is not known, } B = 1.1 \text{ used)}$$

$$C = 1.7 - r_m \text{ (If } r_m \text{ is not known, } C = 0.7 \text{ used); } r_m = M_{o1} / M_{o2};$$

$$M_{o1} \text{ and } M_{o2} \text{ are the first order end moments; } |M_{o2}| \geq |M_{o1}|$$

## 4. RESULTS AND DISCUSSIONS

### 4.1. Two-Dimensional Frame for the Research

The frame shown in the figure (1) below is composed of members with rectangular cross-sections for a beam with a span length of 5500mm and square cross-section for columns with 200mm\*400mm and 280mm\*280mm, respectively. All members are constructed of the same strength concrete ( $E$  is the same for both beams and columns). Materials assumed to be taken C-25, S-300. They are considering bending in the plane of the frame only. Base nodes of the columns are taken as fully rigid. i.e., these ends have zero theoretical relative stiffness.

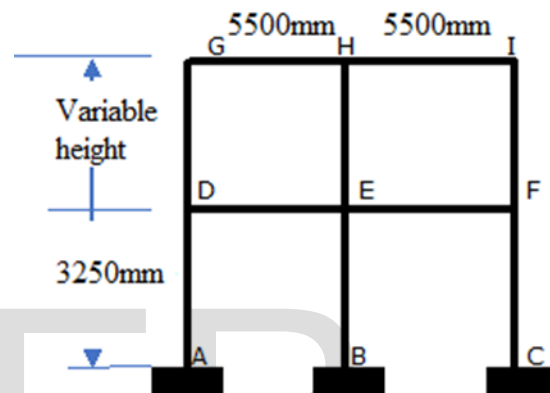


Fig. (1a) Two-dimensional frame indicator at fixed height for the first story with a varying height of the column in the second story

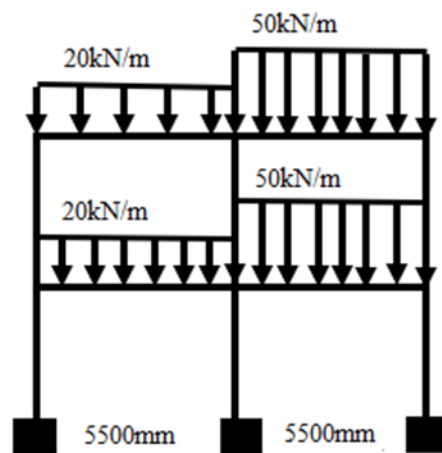


Fig. (1b) Two-dimensional frame indicator at fixed height for the first story with a varying height of the column in the second story with distributed load for the analysis case

When the height of the column increases in the analysis portion, the axial force increases with decreasing first order end moments for interior and exterior unbraced columns, as shown

in table (1) below. In the case braced frames, as shown in table (2), the height of the column increases with a decreasing of end moments for interior and exterior columns of EH, DG, and FI. The axial load varies at different lengths of the column. In some restricted height, the axial load increases and decreases for the interior and exterior columns, as shown in table (2).

1992-1-1:2015 provides the highest effective length factor in the unbraced system of the column, whatever the position of the column located. Again the length of the column increases, the effective length factor decreases irrespective of column position and classification.

Table 1: Force resultants in the 2nd story columns for the unbraced frame

Frame No.	Column Height m	l/h	Column DG			Column EH			Column FI		
			Axial Force kN	Bottom Moment kNm	Top Moment kNm	Axial Force kN	Bottom Moment kNm	Top Moment kNm	Axial Force kN	Bottom Moment kNm	Top Moment kNm
1	2	7.14	55.014	-28.78	32.03	225.801	-44.52	50.53	137.945	70.93	-84.93
2	4	14.28	55.27	-22.03	24.06	237.06	-33.80	38.93	138.187	53.65	-65.26
3	6	21.43	56.85	-17.33	18.90	245.713	-27.04	31.42	127.957	42.35	-52.34
4	8	28.57	59.25	-14.26	15.59	252.718	-22.51	26.31	142.072	35.00	-43.67
5	10	35.71	62.125	-12.12	13.30	258.767	-19.28	22.63	144.908	29.84	-37.49
6	12	42.86	65.293	-10.56	11.64	264.223	-16.85	19.84	148.044	26.03	-32.87
7	14	50.00	68.653	-9.36	10.37	269.289	-14.97	17.67	151.378	23.09	-29.29
8	16	57.14	72.145	-8.42	9.38	274.085	-13.47	15.93	154.849	20.77	-26.43
9	18	64.28	75.734	-7.66	8.58	278.686	-12.23	14.50	158.420	18.88	-24.10
10	20	71.42	79.394	-7.04	7.93	283.143	-11.21	13.30	162.063	17.31	-22.16

Table 2: Force resultants in the 2nd story columns for the braced frame

Frame No.	Column Height m	l/h	Column DG			Column EH			Column FI		
			Axial Force kN	Bottom Moment kNm	Top Moment kNm	Axial Force kN	Bottom Moment kNm	Top Moment kNm	Axial Force kN	Bottom Moment kNm	Top Moment kNm
1	2	7.14	55.499	-24.78	27.72	226.717	-36.30	40.87	137.066	73.75	-86.97
2	4	14.28	59.688	-17.05	18.98	196.662	-25.17	29.75	138.504	55.81	-66.83
3	6	21.43	59.291	-12.90	14.32	172.019	-19.64	23.57	134.728	44.60	-54.04
4	8	28.57	58.506	-10.45	11.57	158.527	-16.16	19.62	131.228	37.26	-45.53
5	10	35.71	58.632	-8.85	9.79	152.593	-13.76	16.85	129.466	32.10	-39.47
6	12	42.86	59.630	-7.73	8.56	150.937	-11.99	14.78	129.190	28.26	-34.91
7	14	50.00	61.272	-6.90	7.65	151.662	-10.63	13.18	129.960	26.29	-31.35
8	16	57.14	63.37	-6.26	6.96	153.744	-9.65	11.89	131.446	22.92	-28.60
9	18	64.28	65.797	-5.76	6.42	156.628	-8.67	10.84	133.430	20.98	-26.16
10	20	71.42	68.465	-5.35	5.98	160.001	-7.94	9.96	135.771	19.37	-24.20

#### 4.2. Comparing Effective Length Factor using EBCS 2: 1995 and ES EN 1992-1-1:2015

The length of a column in the second story was varied from 2000mm to 20000mm with an increment of 2000mm. An increment of 2000mm gives a range for l/h from 14 to 71.42. Where l is the height of the column; and h is the width of the column in the plane of bending. Fig (2a, 2b, and 2c) shows a plot of l/h ratio versus effective length factor for unbraced & braced of an internal and external column. When comparing the codes of practice in terms of effective length factor for braced edge column and inner column in EBCS 2: 1995 is more than that of ES EN 1992-1-1:2015. The unbraced in ESEN 2:2015 is greater than EBCS 2: 2015, as shown in fig. (2). It was visible that ES EN

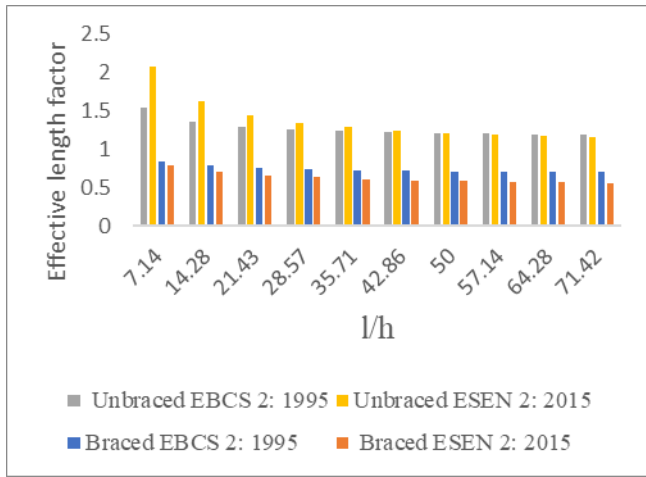


Fig. (2a) effective length factor for unbraced and braced edge column DG

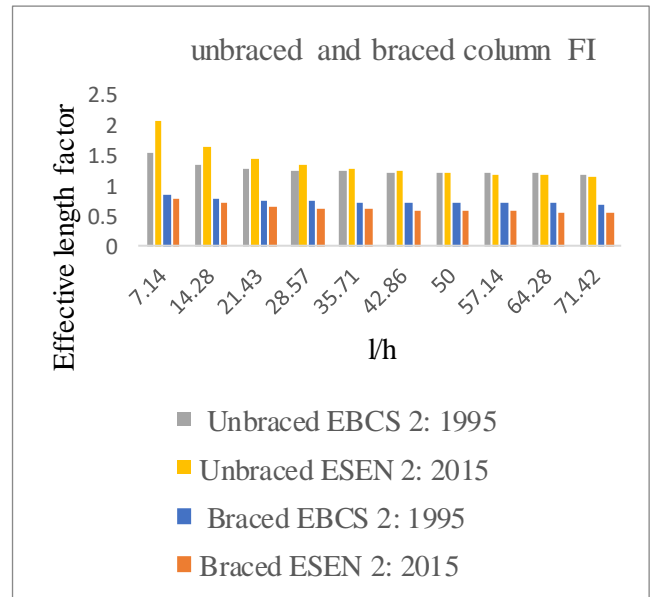


Fig. (2c) effective length factor for unbraced and braced edge column FI

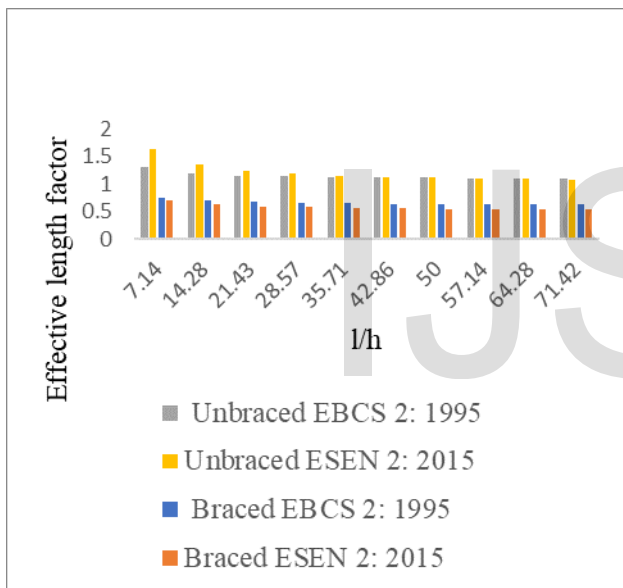


Fig. (2b) effective length factor for unbraced and braced internal column EH

For unbraced column EH, the effective length of EBCS 2: 1995 is greater than ES EN 1992-1-1:2015. i.e., The distance between successive inflection points or points of zero moments is greater in EBCS 2: 1995 than that of ES EN 1992-1-1:2015. For braced column EH, the effective length of ES EN 1992-1-1:2015 is more significant than the code of practiced EBCS 2:1995. The effective length factor is more at the initial l/h values. When the height of the column increases, the effective length factor decreases, as shown in fig(2).

#### 4.3. Comparing slenderness limitation using EBCS 2: 1995 and ES EN 1992-1-1:2015

A higher slenderness ratio indicates lower critical stress that will cause buckling no more crushing. The restriction of slenderness values of ES EN 1992-1-1:2015 is more than the code of practiced EBCS 2: 1995, as shown in table (3a, 3b, and 3c) whatever the type of column, the position of the column, etc.

Table (3a) slenderness limitation for braced and unbraced edge column DG

	DG			
	unbraced EBCS 2:1995	unbraced ESEN 2: 2015	braced EBCS 2:1995	braced ESEN 2: 2015
l/h	slenderness limit	slenderness limit	slenderness limit	slenderness limit
7.14	60.27366969	179.808104	72.34848485	178.7042763
14.28	60.13391966	180.5713637	72.45785037	172.6099479
21.43	59.29239741	178.1331601	72.52094972	173.3550955
28.57	58.07912542	174.3385965	72.57994814	174.67256
35.71	56.71932378	170.0347318	72.59959142	174.5374389
42.86	55.32621162	165.6004385	72.57593458	173.0078064
50	53.95534644	161.2114841	72.54901961	170.6032961
57.14	52.63336377	156.9625269	72.48563218	167.5919619
64.28	51.371092	152.910355	72.42990654	164.3309712
71.42	50.17303861	149.0559067	72.36622074	160.9392525

Table (3b) slenderness limitation for braced and unbraced internal column EH

EH				
	unbraced EBCS 2:1995	unbraced ESEN 2: 2015	braced EBCS 2:1995	braced ESEN 2: 2015
l/h	slenderness limit	slenderness limit	slenderness limit	slenderness limit
7.14	29.75098266	88.15622735	72.20455102	88.22069511
14.28	29.03588752	85.60943515	71.1512605	93.18037915
21.43	28.52004314	83.8388075	70.83156555	99.13104971
28.57	28.12199699	82.50629605	70.59123344	102.8715037
35.71	27.79136	81.42133305	70.41543027	104.560477
42.86	27.50292824	80.49193611	70.28078484	104.9075088
50	27.24300051	79.66566887	70.16312595	104.4603212
57.14	27.00359658	78.91523565	70.2901598	103.9609383
64.28	26.7797593	78.19572907	69.99538745	102.5158352
71.42	26.56815099	77.55981184	69.92971888	101.3229256

Table (3c) slenderness limitation for braced and unbraced edge column FI

FI				
	unbraced EBCS 2:1995	unbraced ESEN 2: 2015	braced EBCS 2:1995	braced ESEN 2: 2015
l/h	slenderness limit	slenderness limit	slenderness limit	slenderness limit
7.14	38.06372294	110.7821556	71.19983902	111.6994812
14.28	38.03037879	110.1148161	70.87759988	110.5560009
21.43	39.52139091	113.8437746	70.63286454	111.6617048
28.57	37.50679875	107.7104836	70.459038	112.8294624
35.71	37.13796171	106.4159266	70.33189764	113.3652635
42.86	36.7425116	105.1124632	70.23775423	113.3162542
50	36.33564416	103.7990041	70.96491228	114.2895501
57.14	35.92609738	102.527015	70.03496503	111.9765066
64.28	35.5188782	101.2651025	70.04969419	111.1670646
71.42	35.11739627	100.0291411	70.01033058	110.1351576

#### 4.4. Comparing column classification using EBCS 2: 1995 and ES EN 1992-1-1:2015

The third step was to study the column classification by considering the two codes, such as EBCS 2: 1995 and ES EN 1992-1-1:2015. For classification purposes, the same frame was selected as the previous one loaded uniformly distributed load of 20kN/m for beam DE and GH, as shown fig.(1). Again 50kN/m load was applied for beam EF and HI. Calculation of limiting slenderness ratios according to ES EN 1992-1-1:2015 involves factor estimations A, B, C, and n as discussed section 3.6. However, for the braced column, estimation of slender-

ness ratio based on EBCS 2: 2015 was taken by calculating the ultimate design end moments for the individual column, as discussed in section 3.5 above.

Table (4a) column classification for unbraced and braced column DG

column DG				
l/h	unbraced EBCS 2:1995	unbraced ESEN 2:2015	braced EBCS 2:1995	braced ESEN 2:2015
7.14	Short	Short	Short	Short
14.28	Slender	Short	Short	Short
21.43	Slender	Short	Short	Short
28.57	Slender	Short	Slender	Short
35.71	Slender	Short	Slender	Short
42.86	Slender	Slender	Slender	Short
50	Slender	Slender	Slender	Short
57.14	Slender	Slender	Slender	Short
64.28	Slender	Slender	Slender	Short
71.42	Slender	Slender	Slender	Short

Table (4b) column classification for unbraced and braced column EH

column EH				
l/h	unbraced EBCS 2:1995	unbraced ESEN 2:2015	braced EBCS 2:1995	braced ESEN 2:2015
7.14	Slender	Short	Short	Short
14.28	Slender	Short	Short	Short
21.43	Slender	Slender	Short	Short
28.57	Slender	Slender	Short	Short
35.71	Slender	Slender	Slender	Short
42.86	Slender	Slender	Slender	Short
50	Slender	Slender	Slender	Short
57.14	Slender	Slender	Slender	Slender
64.28	Slender	Slender	Slender	Slender
71.42	Slender	Slender	Slender	Slender

Table (4c) column classification for unbraced and braced column FI

column FI				
l/h	unbraced EBCS 2:1995	unbraced ESEN 2:2015	braced EBCS 2:1995	braced ESEN 2:2015
7.14	Slender	Short	Short	Short
14.28	Slender	Short	Short	Short
21.43	Slender	Short	Short	Short
28.57	Slender	Slender	Slender	Short
35.71	Slender	Slender	Slender	Short
42.86	Slender	Slender	Slender	Short
50	Slender	Slender	Slender	Short
57.14	Slender	Slender	Slender	Slender
64.28	Slender	Slender	Slender	Slender
71.42	Slender	Slender	Slender	Slender

## 5. CONCLUSIONS

When the height of the column increases in the analysis portion, the axial force increases with decreasing first order end moments for interior and exterior unbraced columns.

According to EBCS 2: 1995 and ES EN 1992-1-1:2015, calculation of slenderness in the design of column have been discussed and compared.

A summary equation to calculate the slenderness status are discussed in section 3.4, 3.5, and 3.6.

The effective length factor variation has been compared and reported according to the old code of practiced EBCS 2:1995 and the new code of practice ES EN 1992-1-1:2015.

In the case of column classification, the two codes showed a somewhat similar pattern. For instance, at 2000mm height of column unbraced EBCS 2: 1995 and unbraced ES EN 1992-1-1:2015 are of the same status. Deviations are there; for example, at l/h greater than 28.57, braced EBCS is slenderer while Braced ESEN is short for edge column DG; the others are discussed briefly through table (4a, 4b, and 4c).

A higher slenderness ratio indicates lower critical stress that will cause buckling no more crushing. The restriction of slenderness values of ES EN 1992-1-1:2015 is more than the code of practiced EBCS 2: 1995.

## 6. ACKNOWLEDGEMENT

I would like to thank my wife Finot. She gave me care, care of our child and help, discussed ideas and prevented several wrong turns.

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